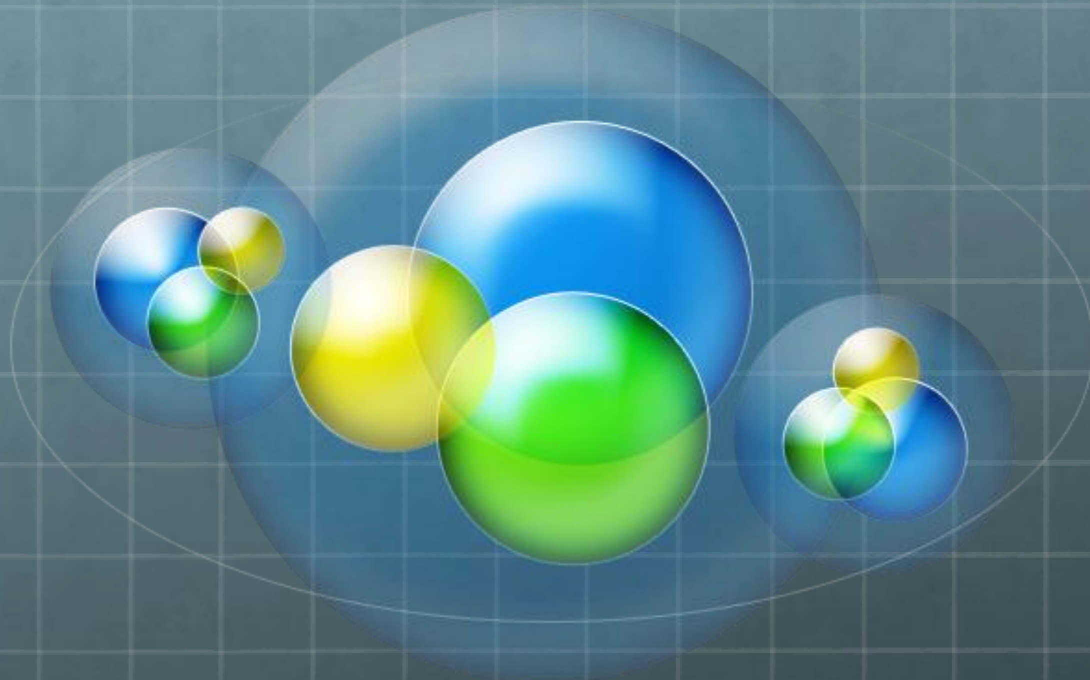

# The Value of Values *and* Their Valuations

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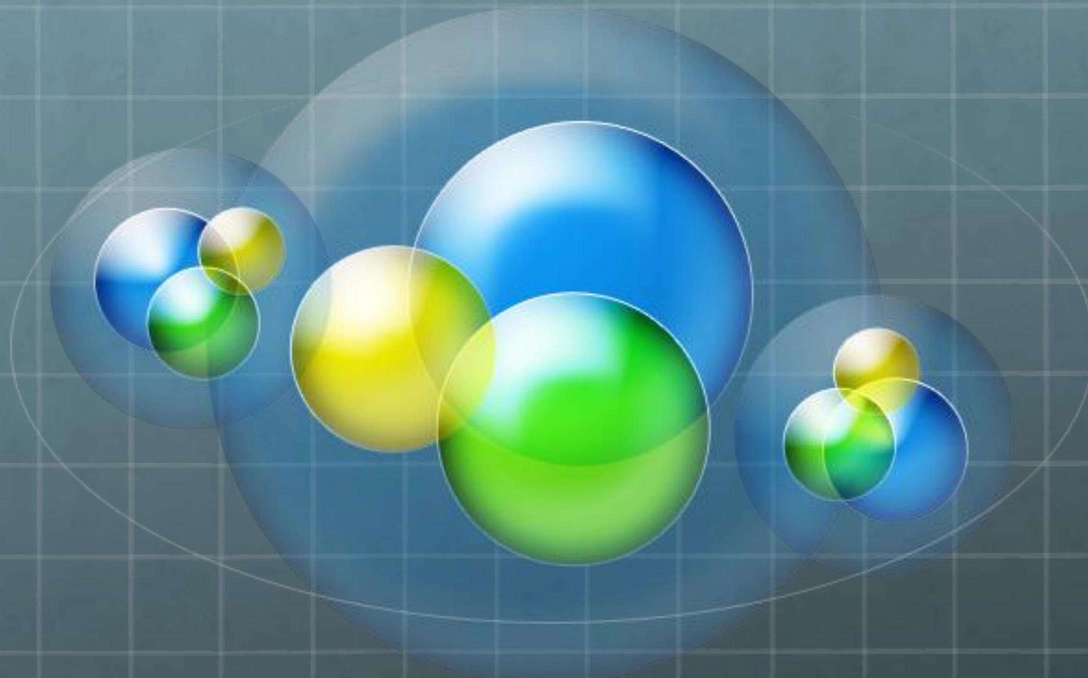


# Why bother?

## *It empowers YOU!*

# Before we start…

- Please MUTE your computer microphone during presentation.
- SO, HOW CAN YOU ASK QUESTIONS?
- The best way is to use the IM feature on the right side of your GoToMeeting screen.
- Just shoot me a question, and I'll respond immediately.
- Or you can just UN-MUTE your mic and ask away !
- *Any questions before we start ?*
- Then let's begin…

# First, the Logic

- Would you be willing to lend me $1,000 today if I promised to repay your $1,000 in a year?
- Okay, so how much are you willing to lend me if I give you $1,000 in a year?
- How about lending me $500?  $100?  $50?  $10?
- Why so little?  Don't you trust me?

# A Fair Return *plus* Uncertainty

- You deserve to be paid a fair return
- You want to be repaid on a timely basis
- You don't know what the future will bring
- How much do you trust the borrower?
- Can you get your money back sooner?

# Next, some lingo…

- **Present Value** – current worth of a future sum (or stream) of money
- **Future Value** – what a sum or stream of payments will be worth at a specific point in the future.
- **Required Return (aka Discount Rate)** – a percentage rate that reflects a lender's required (annual) return
- **Risk Premium** – a quantification in percentage terms of a subjective evaluation of uncertainty.
- **Liquidity Premium** – an adjustment compensating lenders for limited access to their money.

# 4 Symbols & Synonyms

- **Present Value** = **PV** = Net Present Value = NPV = Discounted Present Value = Price you're willing to pay today.
- **Future Value** = **FV** = Value of a sum or stream in tomorrow's terms.
- **Discount Rate** = **Required Rate of Return** = **r** = RRR = the return you want to get from an investment.
- **Time Periods** = **n** = Number of periods (typically years) over which the calculation is considered.

# from Theory to Practice

- **Example A:** If your required return is 7.5% how much would a payment of $1000.00 **in one year** be worth today?
- **Example B:** If your required return is 7.5% how much would a payment of $1000.00 **in five years** be worth today?
- **Example C:** If your required return is 7.5% how much would a payment of $1000.00 **in ten years** be worth today?

# Basic Formula…

$$PV = \frac{FV}{(1+r)^n}$$

# Formula with some inputs…

$$PV = \frac{\$1000}{(1.075)^n}$$

# Formulas with all the numbers…

A. $PV = \frac{\$1000}{(1.075)^1} = \frac{\$1000}{(1.075)} = \$930.23$

B. $PV = \frac{\$1000}{(1.075)^5} = \frac{\$1000}{(1.436)} = \$696.56$

C. $PV = \frac{\$1000}{(1.075)^{10}} = \frac{\$1000}{(2.061)} = \$485.19$

# Logic Check:

***The longer you have to wait to get repaid… The less you're willing to lend.***

# Now let's go the other way:

***What's $1000 today going to be worth in the future?***

# We use the same formula but switch things around

$$FV = PV \times (1+r)^n$$

# Here are the Future Values*

- In 1 year = $1,075.00
- In 5 years = $1,435,63
- In 10 years = $2,061.03
- In 100 years = $1,383,077.21

* 7½ % Required Return

# Here's a fun fact…

- Indians sold Manhattan to Dutch for \$24 in 1626
- If invested at 7½% per year
- Over that time period (389 years)
- The value today would be ???

# $40 Trillion !

*2010 estimate NYC property value*

**$ 1.3 Trillion**

# Now, back to work…

## What about calculating the Present Value of a *stream of payments* over time?

# For example…

**What's the value of $75 paid at end of each year over the next 5 years?**

# It's actually just the sum of 5 PV calculations:

$$PV = \frac{PMT}{(1+r)^1} + \ldots + \frac{PMT}{(1+r)^5}$$

*but it's frequently written as…*

$$PV = \sum_{n=1}^{5} \frac{PMT}{(1+r)^n}$$

# The Stigma of the Sigma…Σ

*It's no big deal*

**Σ = Sigma = Sum**

# Just add them together

$$\frac{\$75}{(1.075)} + \frac{\$75}{(1.156)} + \frac{\$75}{(1.242)} + \frac{\$75}{(1.335)} + \frac{\$75}{(1.435)}$$

$$\$69.77 + \$64.90 + \$60.37 + \$56.16 + \$52.24$$

$303.44

# Why is this important?

**These are the formulas used to price BONDS in the market,**

**and**

**They are used by Actuaries for determining Pension Fund valuations!**

Bonds pay
$1000 at maturity
together with a
stream of payments
called “coupons”.

# Valuing $1000 in 5 years

*Remember this?*

A. $$PV = \frac{\$1000}{(1.075)^1} = \frac{\$1000}{(1.075)} = \$930.23$$

B. $$PV = \frac{\$1000}{(1.075)^5} = \frac{\$1000}{(1.436)} = \$696.56$$

C. $$PV = \frac{\$1000}{(1.075)^{10}} = \frac{\$1000}{(2.061)} = \$485.19$$

# Add that to the coupons flow…

$$\frac{\$75}{(1.075)} + \frac{\$75}{(1.156)} + \frac{\$75}{(1.242)} + \frac{\$75}{(1.335)} + \frac{\$75}{(1.435)}$$

$69.77 + $64.90 + $60.37 + $56.16 + $52.24

**$303.44**

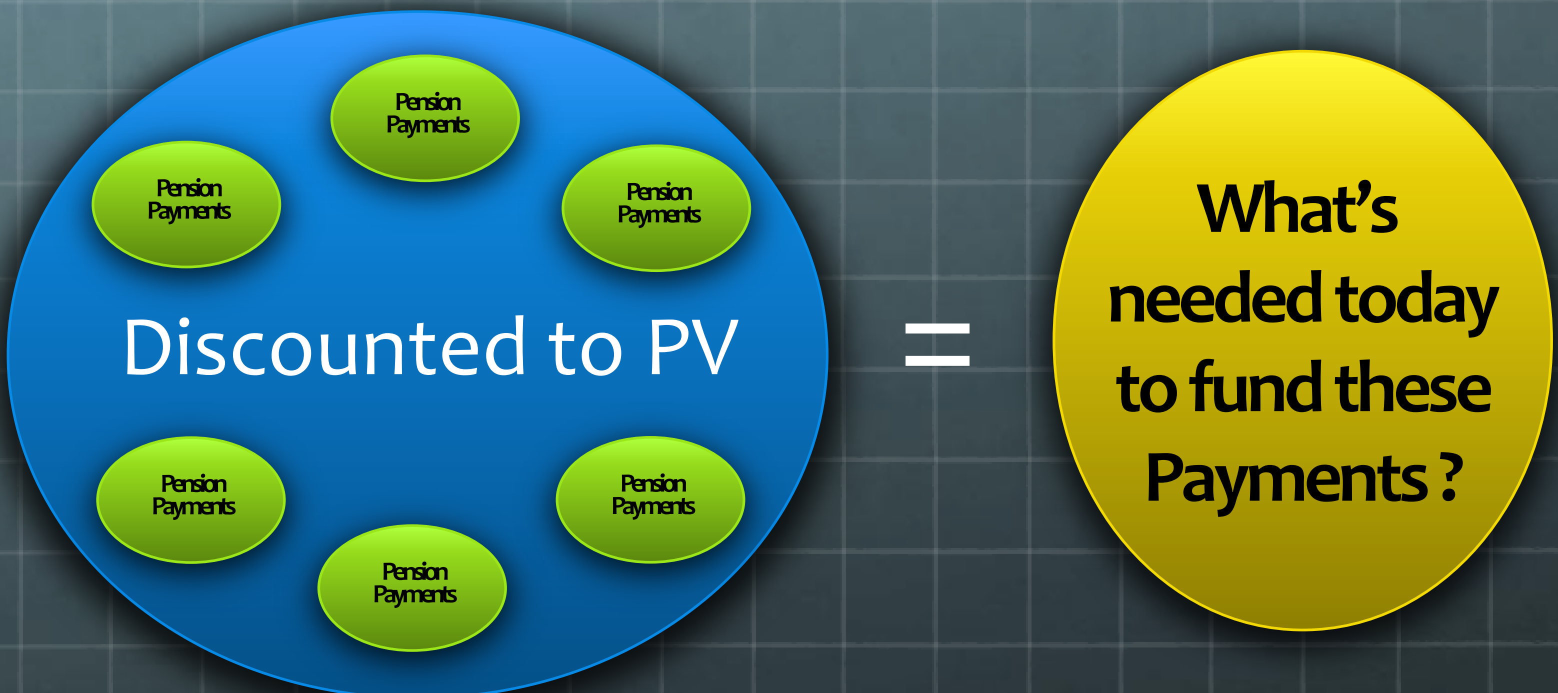
# That's how bonds are priced…

$696.56 + $303.44 = $1,000.00

PRINCIPLE + COUPON = TOTAL

# *Let's discuss Pension Liabilities*

# Actuaries use similar equations to calculate pension fund valuations.



# Meet Dan…

- He’s a 40 year old county administrator
- Been working for 10 years
- And plans to  stay until retirement
- He qualifies for pension benefits under the county’s plan

# Now here's Chuck…

- He's your actuary
- Chuck helps figure out how much you need to set aside for Dan's retirement benefits
- How much does Chuck recommend you have invested to fund these payments?

# Let's help Chuck figure out what's needed…

- Dan will get $20,000 per year  in a lump sum.
- Payments will come at BEGINNING of each year.
- In nominal terms that comes to $500,000.
- Let's make some assumptions:
  - Dan will retire at 65 and live until he's 90
  - His annual payments will not change
  - There are no survivor benefits
  - Annual investment returns will be 7½ %
  - Our goal is to be fully funded

- How much do we recommend should invested today to fund these benefits?

# Let's recall this equation:
## *The PV of a Stream of Payments*

$$PV_{bp} = \frac{\$20{,}000}{(1.075)^{0}} + \ldots + \frac{\$20{,}000}{(1.075)^{24}}$$

Payments made at START of each year!

$$PV_{bp} = \$239{,}659.34$$

# Here are the numbers…

| Year | Payment | Disc Value |
|---|---|---|
| 1 | $ 20,000.00 | $ 20,000.00 |
| 2 | $ 20,000.00 | $ 18,604.65 |
| 3 | $ 20,000.00 | $ 17,306.65 |
| 4 | $ 20,000.00 | $ 16,099.21 |
| 5 | $ 20,000.00 | $ 14,976.01 |
| 6 | $ 20,000.00 | $ 13,931.17 |
| 7 | $ 20,000.00 | $ 12,959.23 |
| 8 | $ 20,000.00 | $ 12,055.10 |
| 9 | $ 20,000.00 | $ 11,214.04 |
| 10 | $ 20,000.00 | $ 10,431.67 |
| 11 | $ 20,000.00 | $ 9,703.88 |
| 12 | $ 20,000.00 | $ 9,026.86 |
| 13 | $ 20,000.00 | $ 8,397.08 |
| 14 | $ 20,000.00 | $ 7,811.24 |
| 15 | $ 20,000.00 | $ 7,266.27 |
| 16 | $ 20,000.00 | $ 6,759.32 |
| 17 | $ 20,000.00 | $ 6,287.74 |
| 18 | $ 20,000.00 | $ 5,849.06 |
| 19 | $ 20,000.00 | $ 5,440.99 |
| 20 | $ 20,000.00 | $ 5,061.38 |
| 21 | $ 20,000.00 | $ 4,708.26 |
| 22 | $ 20,000.00 | $ 4,379.78 |
| 23 | $ 20,000.00 | $ 4,074.21 |
| 24 | $ 20,000.00 | $ 3,789.97 |
| 25 | $ 20,000.00 | $ 3,525.55 |
| Total | $500,000.00 | $239,659.34 |

| Year | Payment | Disc Value |
|---|---|---|
| 1 | $ 20,000.00 | $ 20,000.00 |
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| 24 | $ 20,000.00 | $ 3,789.97 |
| 25 | $ 20,000.00 | $ 3,525.55 |
| Total | $500,000.00 | $239,659.34 |

# But we're not done yet.

# We need to discount this number

# Back to Basics...

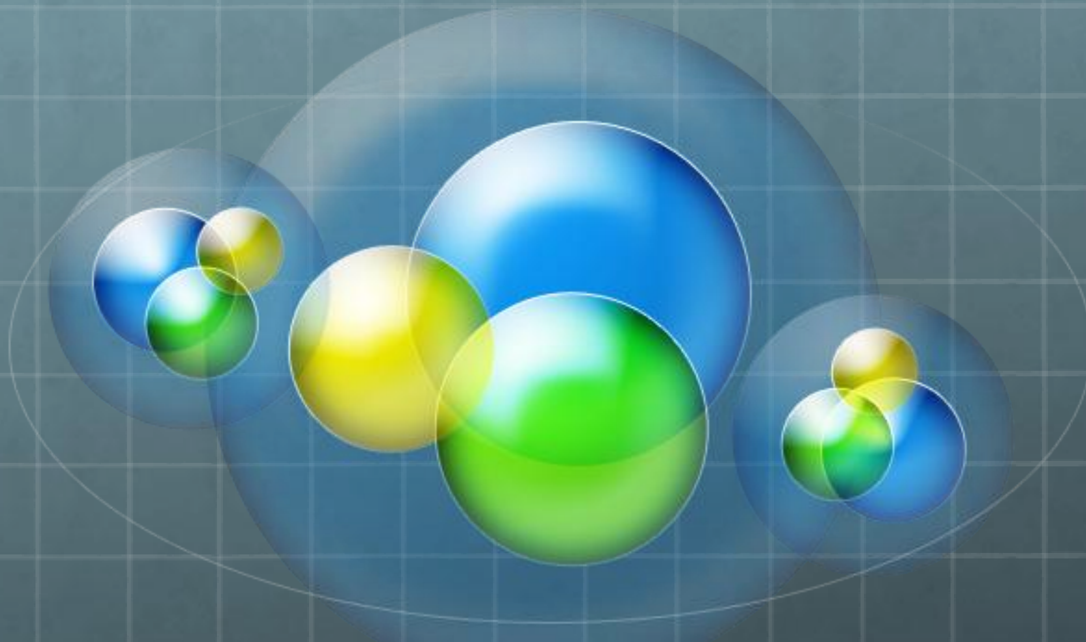
$$PV = \frac{FV}{(1+r)^n}$$

# Now with numbers…

$$PV = \frac{\$239{,}659.34}{(1.075)^{25}}$$

# What’s needed to fund Dan’s benefits…

$$\$39{,}299.11 = \frac{\$239{,}659.34}{6.0983}$$



# You are now empowered…

**Go forth and calculate!**

# This ends the official presentation.

*But if we have more time…*

# What about shorter compounding periods?

- How about monthly, quarterly, semi-annual?
- Changing the compounding period changes the outcome.
- 7½% per year is not 7½% per year compounded monthly!
- Let’s compare the results…

# We saw this earlier…
## *Based on 7½% per year*

A. $PV = \dfrac{\$1000}{(1.075)^1} = \dfrac{\$1000}{(1.075)} = \$930.23$

B. $PV = \dfrac{\$1000}{(1.075)^5} = \dfrac{\$1000}{(1.436)} = \$696.56$

C. $PV = \dfrac{\$1000}{(1.075)^{10}} = \dfrac{\$1000}{(2.061)} = \$485.19$

# Now consider 7½% per year compounded monthly

A. $PV = \dfrac{\$1000}{(1.00625)^{12}} = \dfrac{\$1000}{(1.07763)} = \$927.96$

B. $PV = \dfrac{\$1000}{(1.00625)^{60}} = \dfrac{\$1000}{(1.45329)} = \$688.09$

C. $PV = \dfrac{\$1000}{(1.00625)^{120}} = \dfrac{\$1000}{(2.11206)} = \$473.47$

# The shorter the compounding period the greater the impact

- 7.5% compounded annually = 7.500%
- 7.5% compounded semi-annually = 7.641%
- 7.5% compounded monthly = 7.763%
- 7.5% compounded weekly = 7.783%
- 7.5% compounded daily = 7.788%
- 7.5% compounded continuously ???

# There is a special formula for continuous compounding...

- To calculate use Napier's number which is based on a natural logarithm)
- Napier's number = *e* = ~ 2.7183
- The formula is: $e^{(r)}$
- So it is: $2.7183^{(.075)}$ = 1.7789 or 7.789%
- Which is 3.9% greater than the annual rate!

# Thanks for participating!